17216

21314

3 Hours/100 Marks

Seat No. | | | | | | | | |

Instructions : *(1)* ***All*** *questions are* ***compulsory****.*
(2) Illustrate your answers with neat sketches ***wherever*** *necessary.*
(3) Figures to the ***right*** *indicate* ***full*** *marks.*
(4) Assume suitable data, if ***necessary****.*
(5) Use of Non-programmable Electronic Pocket Calculator is ***permissible****.*
(6) Mobile Phone, Pager and any other Electronic Communication devices are ***not permissible*** *in Examination Hall.*

MARKS

1. Attempt **any ten** of the following : **20**

a) If $f(x) = \cos x$, show that $f(3x) = 4f^3(x) - 3f(x)$.

b) Express in the form a + ib, $\frac{1+i}{2-i}$ where $a, b \in R$, $i = \sqrt{-1}$.

c) Evaluate $\lim_{x \to 0} \frac{1}{\sqrt{x+1}-1}$.

d) Evaluate $\lim_{x \to 0} \frac{2^x - 1}{\sin 2x}$.

e) If $f(x) = 3x^2 - 5x + 7$ show that $f(-1) = 3f(1)$.

f) Find x and y, if $x[1 - i] + y[2 + i] + 6 = 0$.

g) Evaluate $\lim_{x \to 0} \frac{1 - \cos 2x}{x^2}$.

h) If $y = \cos^{-1}(\sin x)$ find $\frac{dy}{dx}$.

i) If $y = e^x \cdot \sin x \cdot \cos x \cdot$ find $\frac{dy}{dx}$.

P.T.O.

MARKS

j) Find $\frac{dy}{dx}$ if $y = x^x$.

k) Find first two real roots of equation, $x^3 - 2x - 5 = 0$ using bisection method.

l) Find the first iteration by using Jacobi's method for the following system of equation

$5x - y + z = 10$, $x + 2y = 6$, $x + y + 5z = -1$.

2. Attempt **any four** of the following : **16**

a) If $f(x) = \frac{x-4}{4x-1}$ then show that f [f(x)] = x.

b) If $f(x) = \log\left[\frac{1+x}{1-x}\right]$ then show that $f(a) + f(b) = f\left[\frac{a+b}{1+ab}\right]$.

c) Using Euler's formula prove that $\cos 2\theta = \cos^2\theta - \sin^2\theta$.

d) Simplify using De Moivre's theorem :

$$\frac{(\cos 5\theta - i\sin 5\theta)^{2/5} \cdot (\cos \frac{2}{7}\theta + i\sin \frac{2}{7}\theta)^2}{(\cos 4\theta + i\sin 4\theta)^{1/4} \ [\cos \frac{2}{3}\theta - i\sin \frac{2}{3}\theta]^3}$$

e) Find cube-roots of unity.

f) Simplify $1 + i^{100} + i^{10} + i^{50}$.

3. Attempt **any four** of the following : **16**

a) If $f(x) = ax^2 + bx + 3$ and f(1) = 4, f(2) = 11, find 'a' and 'b'.

b) If f(x) = sin x, g(x) = cos x prove that :

i) f(x + y) = f(x) g(y) + g(x) f(y).

ii) g(m – n) = g(m) · g(n) + f(m).f(n).

c) Evaluate $\lim_{x \to \pi/4} \frac{\sin^2 x - \cos^2 x}{1 - \tan x}$.

d) Evaluate $\lim_{x \to \infty} x\left[\sqrt{x^2+1} - \sqrt{x^2-1}\right]$.

MARKS

e) Evaluate $\lim_{x \to 4} \frac{x^2 - 7x + 12}{x^3 - 64}$.

f) Evaluate $\lim_{x \to 0} \frac{a^x + a^{-x} - 2}{\sin^2 x}$.

4. Attempt **any four** of the following : **16**

a) If u and v are differentiable functions of x and $y = u/v$, then prove that

$$\frac{dy}{dx} = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}.$$

b) If $y = \sin^{-1}[3x - 4x^3]$ find dy/dx.

c) Find $\frac{dy}{dx}$, if $13x^2 + 2x^2y + y^3 = 1$.

d) Find the derivative of (x) $\sin^{-1} x$.

e) Using first principle find derivative of $f(x) = a^x$.

f) Find dy/dx if $y = \log\left[x + \sqrt{x^2 + a^2} \right]$.

5. Attempt **any four** of the following : **16**

a) Evaluate $\lim_{x \to 1} \frac{\sin \pi x}{x - 1}$.

b) Evaluate $\lim_{x \to 0} \frac{x}{\sqrt{9 - x + x^2} - 3}$.

c) Using Bisection method find the approximate root of $x^2 + x - 3 = 0$ [carry out three iteration].

d) Using Newton-Raphson method find approximate value of $\sqrt[3]{100}$ (perform three iterations).

MARKS

e) Using Regula Falsi method find approximate root of equation $x^3 + 2x^2 - 8 = 0$ (take three iteration).

f) Find the real root of the equation $x \cdot e^x = 3$ using False position method (Two iterations only).

6. Attempt **any four** of the following : **16**

a) Differentiate $\cos^{-1}[2x^2 - 1]$ w.r.t. $\sin^{-1}\left[2x \cdot \sqrt{1 - x^2}\right]$.

b) If $y = \sin 5x - 3\cos 5x$, show that $\frac{d^2y}{dx^2} + 25y = 0$.

c) Solve the following equations by Jacobi's method, performing three iterations only :

$20x + y - 2z = 17$, $3x + 20y - z = -18$ and $2x - 3y + 20z = 25$.

d) Solve the following equations by Gauss-Seidal method taking three iterations only :

$15x + 2y + z = 18$, $2x + 20y - 3z = 19$, $3x - 6y + 25z = 22$.

e) Solve the following equations by Gauss elimination method,

$x + 2y + 3z = 14$, $3x + y + 2z = 11$ and $2x + 3y + z = 11$.

f) With the following system of equations $5x - y = 9$, $x - 5y + z = -4$, $y - 5z = 6$ set up the Gauss-Seidal iterations scheme for solution. Iterate two times, using initial approximations as $x_0 = 1.5$, $y_0 = 0.5$, $z_0 = -0.5$.